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Combined chiral dynamics and MIT bag model study of strong $\Sigma_Q^{(*)} \to \Lambda_Q \pi$ decays

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Abstract. The strong decays of the heavy baryons $\Sigma_Q^{(*)} \to \Lambda_Q \pi$ are studied by combining the chiral dynamics and the MIT bag model. In the charm sector, we calculate the decay widths $\Gamma(\Sigma_c^{(*)} \to \Lambda_c \pi)$ and compare these with the experimental data and other theoretical estimations. In addition, we also predict the strong decay widths $\Gamma(\Sigma_b^{(*)} \to \Lambda_b \pi)$.

1 Introduction

The two exotic relatives of the proton and neutron $\Sigma_b^{(*)+}$ (buu) and $\Sigma_b^{(*)-}$ (bdd) were discovered by the CDF collaboration at Fermilab [1]. Their masses are $M_{\Sigma_b^+} = 5808^{+2.0}_{-2.3} \pm 1.7\,\mathrm{MeV}$, $M_{\Sigma_b^-} = 5816^{+1.0}_{-1.0} \pm 1.7\,\mathrm{MeV}$, $M_{\Sigma_b^{*+}} = 5829^{+1.6}_{-1.8} \pm 1.7\,\mathrm{MeV}$, and $M_{\Sigma_b^{*-}} = 5837^{+2.1}_{-1.9} \pm 1.7\,\mathrm{MeV}$. Due to the mass of the bottom baryon Λ_b^0, $5624 \pm 9\,\mathrm{MeV}$ [2], the mass differences $M_{\Sigma_b^{(*)}} - M_{\Lambda_b}$ are all larger than M_π; thus the dominant decay modes of these exotic bottom baryons are the strong decays $\Sigma_b^{(*)} \to \Lambda_b \pi$, which are similar to the ones of the charm baryons $\Sigma_c^{(*)} \to \Lambda_c \pi$. In this paper, we will combine the chiral dynamics and the MIT bag model to study the strong decays of the heavy baryon $\Sigma_Q^{(*)} \to \Lambda_Q \pi$.

The light quark contribution to the QCD Lagrangian,

$$\mathcal{L}_q = \bar{q}(\mathrm{i}\not{D} - m_q)q\,, \quad (q = u\,, d\,, \text{and}\, s)\,, \tag{1}$$

has an approximate $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ flavor chiral symmetry, because the current quark masses are all very small on the typical hadron energy scale. It became known early on that this symmetry must be spontaneously broken by the QCD vacuum, so that the physical spectra of the hadrons made up of light quarks have only $\mathrm{SU}(3)_{\mathrm{L+R}}$ symmetry. Moreover, due to spontaneous breaking of the chiral symmetry, there exist eight pseudoscalar bosons (called Goldstone bosons, which include three π, four K, and one η). Their couplings to the hadrons at low energies are determined by partial conservation of the axial-vector current (PCAC) and by the current algebra. The chiral properties of a heavy hadron is dictated by its light quark contents. For the heavy baryon, a heavy quark Q will combine the two light quarks to form baryons, Qq_1q_2. However, the two light quarks can form a symmetric sextet or an antisymmetric antitriplet in flavor SU(3) space. For the ground state baryons, the SU(3) symmetric sextet diquarks have spin 1, whereas the SU(3) antisymmetric antitriplet diquarks have spin 0. Thus for the SU(3) symmetric sextet there are both spin $\frac{1}{2}$ baryons (B_6) and spin $\frac{3}{2}$ baryons (B_6^*). For the SU(3) antisymmetric antitriplet there are only spin $\frac{1}{2}$ baryons ($B_{\bar{3}}$). Once the flavor SU(3) contents of these heavy baryons are determined, their couplings to the Goldstone bosons can immediately be written down following the standard formalism of chiral dynamics.

The MIT bag model [3, 4] is a simple relativistic model of hadrons, which is consistent with some of the essential features of QCD, namely confinement, gluon degree of freedom, and gauge invariance. It has been applied to the description of the hadron spectroscopy [5, 6] and various transitions for which the momentum transfers are not large [6]. The greatest success of the MIT bag model lies in its description of ground state hadrons, formed with light quarks, where the agreement with the experimental data is remarkable.

The paper is organized as follows. In Sect. 2 we review the basic MIT bag model formalism and construct the heavy baryon wave functions. In Sect. 3 we consider the dynamics of heavy baryons interacting with the Goldstone bosons. We discuss the chiral properties of the heavy baryons and derive the chiral Lagrangian involving heavy baryons. Applications and numerical results for the strong decay widths of some heavy baryons are worked out in Sect. 4. Finally, our conclusion is given in Sect. 5.

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2 Formalism of MIT bag model for heavy baryon

The MIT bag model is essentially a relativistic shell model that describes quarks moving independently inside a confining spherical cavity of radius R. The bag Lagrangian for quarks only is

$$\mathcal{L}_{\rm bag} = \left\{ \frac{\mathrm{i}}{2}\left[\bar{\psi}\gamma^\mu\partial_\mu\psi - (\partial_\mu\bar{\psi})\gamma^\mu\psi\right] - m\bar{\psi}\psi - \mathcal{B}\right\}\theta_{\rm V}(r) - \frac{1}{2}\bar{\psi}\psi\Delta_s(r)\,, \tag{2}$$

where m is the quark mass, $\theta_{\rm V}(r)=\theta(R-r)$, $\Delta_s(r)=\delta(R-r)$, the constant $\mathcal{B}$ is the volume energy density, which comes from the work done against the QCD vacuum in creating the cavity, and finally a surface term is added to $\mathcal{L}_{\rm bag}$ so that the quarks move as if they had an infinite mass outside the bag [7].

The Euler–Lagrange equations of motion are

$$(\mathrm{i}\gamma^\mu\partial_\mu - m)\psi = 0 \qquad \text{inside the bag}\,, \tag{3}$$

$$\left.\begin{aligned} \mathrm{i}\gamma^\mu n_\mu\psi &= \psi \\ n_\mu\cdot\partial^\mu(\bar{\psi}\psi) &= 2\mathcal{B} \end{aligned}\right\} \quad \text{on the bag surface}\,, \tag{4}$$

where n_μ is a covariant inward-pointing vector normal to the bag surface, $n_\mu=(0,-\hat{r})$ for a static spherical bag. From (3), we see that the Lagrangian density (2) yields a free Dirac equation inside the bag as expected. The quadratic boundary condition (the second line of (4)) requires that only $j=1/2$ modes can be excited within a static rigid bag. Consequently only the S and P $(l=0,1)$ orbital angular momenta are allowed, and the single particle excitation spectrum consists of two classes of states with total angular momentum $j=1/2$, namely $nS_{1/2}$ and $nP_{1/2}$ (n is the radial quantum number). Explicit expressions for these solutions are [6]

$$u_n^0 = N_n^0 \begin{pmatrix} \sqrt{\left(\frac{\omega+m}{\omega}\right)}\,\mathrm{i}j_0(X_n\frac{r}{R})\,\chi \\ -\sqrt{\left(\frac{\omega-m}{\omega}\right)}\,j_1(X_n\frac{r}{R})\sigma\hat{r}\,\chi \end{pmatrix} \mathrm{e}^{-\mathrm{i}\omega t}\,, \tag{5}$$

and

$$u_n^1 = N_n^1 \begin{pmatrix} \sqrt{\left(\frac{\omega+m}{\omega}\right)}\,\mathrm{i}j_1\left(X_n\frac{r}{R}\right)\sigma\hat{r}\,\chi \\ \sqrt{\left(\frac{\omega-m}{\omega}\right)}\,j_0\left(X_n\frac{r}{R}\right)\,\chi \end{pmatrix} \mathrm{e}^{-\mathrm{i}\omega t}\,, \tag{6}$$

where the normalization coefficients N_n^0, N_n^1 are fixed by the integral $\int_R \psi^\dagger\psi\mathrm{d}^3r=1$, and j_0 and j_1 are the spherical Bessel functions

$$j_0(x)=\frac{\sin x}{x}\,,\quad j_1(x)=\frac{\sin x}{x^2}-\frac{\cos x}{x}\,, \tag{7}$$

χ is a Pauli spinor, X_n is the quark's momentum times R, and ω is the eigenenergy given by

$$\omega=\sqrt{\left(m^2+\frac{X_n^2}{R^2}\right)}\,. \tag{8}$$

The eigenvalue equations can be derived by substituting (5) and (6) into the first line of (4). For u_n^0, we get

$$\begin{pmatrix} 0 & -\mathrm{i}\sigma\hat{r} \\ \mathrm{i}\sigma\hat{r} & 0 \end{pmatrix} \begin{pmatrix} \sqrt{\left(\frac{\omega+m}{\omega}\right)}\,\mathrm{i}j_0(X_n) \\ -\sqrt{\left(\frac{\omega-m}{\omega}\right)}\,j_1(X_n)\sigma\hat{r} \end{pmatrix} = \begin{pmatrix} \sqrt{\left(\frac{\omega+m}{\omega}\right)}\,\mathrm{i}j_0(X_n) \\ -\sqrt{\left(\frac{\omega-m}{\omega}\right)}\,j_1(X_n)\sigma\hat{r} \end{pmatrix}, \tag{9}$$

which leads to

$$j_1(X_n)=\sqrt{\left(\frac{\omega+m}{\omega-m}\right)}\,j_0(X_n)\,, \tag{10}$$

and finally

$$\tan X_n = \frac{X_n}{1-mR-\sqrt{m^2R^2+X_n^2}}\,. \tag{11}$$

The corresponding eigenvalue equation for the u_n^1 mode is

$$\tan X_n = \frac{X_n}{1-mR+\sqrt{m^2R^2+X_n^2}}\,. \tag{12}$$

When $m\to 0$, (11) and (12) yield the transcendental equations

$$\tan X_n = \frac{X_n}{1\mp X_n}\,. \tag{13}$$

Some of the low lying solutions to (13) are listed in Table 1 [8].

After solving the bag equations of motion inside a cavity, we can expand the second quantized light quark field operator $\psi_q(x)$ in terms of the quark eigenmodes,

$$\psi_q(x)=\sum_{nlm} b_{nlm}u_{nlm}(\mathbf{r},t)+d_{nlm}^\dagger v_{nlm}(\mathbf{r},t) \tag{14}$$

where b_{nlm} is the canonical quark annihilation operator and $d_{nlm}^\dagger$ is the antiquark creation operator. These operators satisfy the usual anticommutation relations, namely

$$\left\{b_{nlm}, b^\dagger_{n'l'm'}\right\} = \left\{d_{nlm}, d^\dagger_{n'l'm'}\right\} = \delta_{nn'}\delta_{ll'}\delta_{mm'}\,, \tag{15}$$

with the other ones equal to zero. We can then readily construct a state of the heavy baryon with spin S:

$$\left|B_Q^{nl}\right\rangle = C(S,S_z,s_1,s_2,s_3)\left|q_1^{nl}(s_1)q_2^{nl}(s_2)Q(s_3)\right\rangle\,, \tag{16}$$

Table 1. Bag model eigenmodes for a massless quark

State	$1S_{1/2}$	$1P_{1/2}$	$2S_{1/2}$	$2P_{1/2}$	$3S_{1/2}$	$3P_{1/2}$
X_n	2.043	3.812	5.396	7.002	8.578	10.163

where $C(S, S_z, s_1, s_2, s_3)$ is the familiar Clebsch–Gordan coefficient. Each light quark is in a triplet,

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \tag{17}$$

of the flavor SU(3) group. Since $3 \otimes 3 = 6 \oplus \bar{3}$ and the lowest lying light quark state has $n = 1$ and $l = 0$ (S-wave), there are two different diquarks: a symmetric sextet ($s_1 + s_2 = 1$) and an antisymmetric antitriplet ($s_1 + s_2 = 0$). When the diquark combines with a heavy quark, the sextet contains both spin $\frac{1}{2}$ (B_6) and spin $\frac{3}{2}$ (B_6^*) baryons, and the antitriplet contains only spin $\frac{1}{2}$ ($B_{\bar{3}}$) baryons. Explicitly, the wave functions of the spin $\frac{1}{2}$ B_6 baryons are

$$\begin{aligned}
\left|\Sigma_Q^{+1}\uparrow\right\rangle &= \sqrt{\frac{2}{3}}|uu\rangle|\uparrow\uparrow\rangle|Q\downarrow\rangle \\
&\quad - \sqrt{\frac{1}{3}}|uu\rangle\sqrt{\frac{1}{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)|Q\uparrow\rangle\,, \\
\left|\Sigma_Q^{0}\uparrow\right\rangle &= \sqrt{\frac{2}{3}}\sqrt{\frac{1}{2}}(|ud\rangle + |du\rangle)|\uparrow\uparrow\rangle|Q\downarrow\rangle \\
&\quad - \sqrt{\frac{1}{3}}\sqrt{\frac{1}{2}}(|ud\rangle + |du\rangle) \\
&\quad \times \sqrt{\frac{1}{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)|Q\uparrow\rangle\,, \\
\left|\Sigma_Q^{-1}\uparrow\right\rangle &= \left|\Sigma_Q^{+1}\uparrow\right\rangle_{(u\to d)}\,, \\
\left|\Xi_Q^{\prime +1/2}\uparrow\right\rangle &= \left|\Sigma_Q^{0}\uparrow\right\rangle_{(d\to s)}\,, \\
\left|\Xi_Q^{\prime -1/2}\uparrow\right\rangle &= \left|\Xi_Q^{\prime +1/2}\uparrow\right\rangle_{(u\to d)}\,, \\
\left|\Omega_Q\uparrow\right\rangle &= \left|\Sigma_Q^{+1}\uparrow\right\rangle_{(u\to s)}\,,
\end{aligned} \tag{18}$$

where the superscript denotes the value of the isospin quantum number I_3. An asterisk on the symbol will denote a corresponding spin $\frac{3}{2}$ baryon:

$$\begin{aligned}
\left|\Sigma_Q^{*+1}\frac{1}{2}\right\rangle &= \sqrt{\frac{1}{3}}|uu\rangle|\uparrow\uparrow\rangle|Q\downarrow\rangle \\
&\quad + \sqrt{\frac{2}{3}}|uu\rangle\sqrt{\frac{1}{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)|Q\uparrow\rangle, \\
\left|\Sigma_Q^{*0}\frac{1}{2}\right\rangle &= \sqrt{\frac{1}{3}}\sqrt{\frac{1}{2}}(|ud\rangle + |du\rangle)|\uparrow\uparrow\rangle|Q\downarrow\rangle \\
&\quad + \sqrt{\frac{2}{3}}\sqrt{\frac{1}{2}}(|ud\rangle + |du\rangle) \\
&\quad \times \sqrt{\frac{1}{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)|Q\uparrow\rangle\,, \\
\left|\Sigma_Q^{*-1}\frac{1}{2}\right\rangle &= \left|\Sigma_Q^{+1}\uparrow\right\rangle_{(u\to d)}\,, \\
\left|\Xi_Q^{\prime *+1/2}\frac{1}{2}\right\rangle &= \left|\Sigma_Q^{0}\uparrow\right\rangle_{(d\to s)}\,, \\
\left|\Xi_Q^{\prime *-1/2}\frac{1}{2}\right\rangle &= \left|\Xi_Q^{\prime *+1/2}\frac{1}{2}\right\rangle_{(u\to d)}\,,
\end{aligned}$$

$$\left|\Omega_Q^{*}\frac{1}{2}\right\rangle = \left|\Sigma_Q^{*+1}\frac{1}{2}\right\rangle_{(u\to s)}\,, \tag{19}$$

As to the wave functions of the spin $\frac{1}{2}$ $B_{\bar{3}}$ baryons, they are

$$\begin{aligned}
|\Lambda_Q\uparrow\rangle &= \sqrt{\frac{1}{2}}(|ud\rangle - |du\rangle)\sqrt{\frac{1}{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)|Q\uparrow\rangle, \\
\left|\Xi_Q^{+1/2}\uparrow\right\rangle &= |\Lambda_Q\uparrow\rangle_{(d\to s)}\,, \quad \left|\Xi_Q^{-1/2}\uparrow\right\rangle = \left|\Xi_Q^{+1/2}\uparrow\right\rangle_{(u\to d)}\,.
\end{aligned} \tag{20}$$

We may use the decomposition

$$\begin{aligned}
q_{1i}q_{2j} &= \frac{1}{2}(q_{1i}q_{2j} + q_{1j}q_{2i}) + \frac{1}{2}(q_{1i}q_{2j} - q_{1j}q_{2i}) \\
&= (B_6)_{ij} + \frac{1}{\sqrt{2}}(B_{\bar{3}})_{ij}
\end{aligned} \tag{21}$$

to assemble the sextet and the antitriplet in a systematic and an antisymmetric 3×3 matrix, respectively:

$$B_6 = \begin{bmatrix} \Sigma_Q^{+1} & \frac{1}{\sqrt{2}}\Sigma_Q^{0} & \frac{1}{\sqrt{2}}\Xi_Q^{\prime +1/2} \\ \frac{1}{\sqrt{2}}\Sigma_Q^{0} & \Sigma_Q^{-1} & \frac{1}{\sqrt{2}}\Xi_Q^{\prime -1/2} \\ \frac{1}{\sqrt{2}}\Xi_Q^{\prime +1/2} & \frac{1}{\sqrt{2}}\Xi_Q^{\prime -1/2} & \Omega_Q \end{bmatrix}, \tag{22}$$

$$B_{\bar{3}} = \begin{bmatrix} 0 & \Lambda_Q & \Xi_Q^{+1/2} \\ -\Lambda_Q & 0 & \Xi_Q^{-1/2} \\ -\Xi_Q^{+1/2} & \Xi_Q^{-1/2} & 0 \end{bmatrix}, \tag{23}$$

and a matrix for B_6^*

$$B_6^* = \begin{bmatrix} \Sigma_Q^{*+1} & \frac{1}{\sqrt{2}}\Sigma_Q^{*0} & \frac{1}{\sqrt{2}}\Xi_Q^{\prime +1/2} \\ \frac{1}{\sqrt{2}}\Sigma_Q^{*0} & \Sigma_Q^{*-1} & \frac{1}{\sqrt{2}}\Xi_Q^{\prime *-1/2} \\ \frac{1}{\sqrt{2}}\Xi_Q^{\prime *+1/2} & \frac{1}{\sqrt{2}}\Xi_Q^{\prime *-1/2} & \Omega_Q^{*} \end{bmatrix}. \tag{24}$$

3 Chiral dynamics of heavy baryons

Before discussing the interaction between the heavy baryons and Goldstone bosons, we will first summarize the case of Goldstone bosons interacting among themselves [9–11]. The chiral symmetry is nonlinearly realized by using the unitary matrix

$$\Sigma = \mathrm{e}^{2\mathrm{i}M/\sqrt{2}f_\pi} \tag{25}$$

where M is a 3×3 matrix for the octet of Goldstone bosons:

$$M = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{bmatrix} \tag{26}$$

and $f_\pi = 93\,\mathrm{MeV}$ is the pion decay constant. Under $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$, Σ transforms as

$$\Sigma \to \Sigma' = L\Sigma R^\dagger\,. \tag{27}$$

Hence the lowest order Lagrangian for the Goldstone bosons is

$$\mathcal{L}_M = \frac{f_\pi^2}{4} \mathrm{Tr} \partial_\mu \Sigma^\dagger \partial^\mu \Sigma \,. \tag{28}$$

In order to facilitate the discussions of the Goldstone bosons interacting with the heavy mesons, we introduce a new Goldstone-boson matrix $\xi \equiv \Sigma^{1/2}$, which transforms under an $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ as follows:

$$\xi \to \xi' = L\xi U^\dagger = U\xi R^\dagger \,, \tag{29}$$

where U is an unitary matrix depending on L, R, and M, so that it is no longer global. Now with the aid of ξ, we construct a vector field V_μ and an axial-vector field A_μ:

$$V_\mu = \frac{1}{2} \left(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger \right) \,, \tag{30}$$

$$A_\mu = \frac{\mathrm{i}}{2} \left(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right) \,, \tag{31}$$

with the simple transformation properties

$$V_\mu \to V'_\mu = U V_\mu U^\dagger + U \partial_\mu U^\dagger \,, \tag{32}$$

$$A_\mu \to A'_\mu = U A_\mu U^\dagger \,. \tag{33}$$

The chiral transformation of q is given in (26), which is however not convenient for our purposes. By means of the following field redefinition [12]:

$$q_L \to \xi^\dagger q_L \,, \quad q_R \to \xi q_R \,, \tag{34}$$

the light quarks can be made to transform simply as

$$q \to q' = Uq \,. \tag{35}$$

and the chiral transformations of the heavy baryons in (22) and (23) can be established as

$$B_6 \to B'_6 = U B_6 U^{\mathrm{T}} \,, \tag{36}$$

$$B_{\bar 3} \to B'_{\bar 3} = U B_{\bar 3} U^{\mathrm{T}} \,, \tag{37}$$

and the covariant derivatives under chiral transformations for B_6 and $B_{\bar 3}$ are

$$D_\mu B_6 \equiv \partial_\mu B_6 + V_\mu B_6 + B_6 V_\mu^{\mathrm{T}} \,, \tag{38}$$

$$D_\mu B_{\bar 3} \equiv \partial_\mu B_{\bar 3} + V_\mu B_{\bar 3} + B_{\bar 3} V_\mu^{\mathrm{T}} \,. \tag{39}$$

Similar equations hold for B_6^* and $D_\mu B_6^*$. Then the chiral-invariant Lagrangian is

$$\begin{aligned}
\mathcal{L}_B = {} & \mathrm{tr}[\bar B_6 (\mathrm{i} \not{D} - M_6) B_6] + \frac{1}{2} \mathrm{Tr}[\bar B_{\bar 3} (\mathrm{i} \not{D} - M_{\bar 3}) B_{\bar 3}] \\
& + \mathrm{Tr}\{\bar B_6^{*\mu} [-g_{\mu\nu} (\mathrm{i} \not{D} - M_6^*) + \mathrm{i}(\gamma_\mu D_\nu + \gamma_\nu D_\mu) \\
& - \gamma_\mu (\mathrm{i} \not{D} + M_6^*) \gamma_\nu] B_6^{*\nu}\} \\
& + g_1 \mathrm{Tr}(\bar B_6 \gamma_\mu \gamma_5 A^\mu B_6) + g_2 \mathrm{Tr}(\bar B_6 \gamma_\mu \gamma_5 A^\mu B_{\bar 3}) + \mathrm{H.c.} \\
& + g_3 \mathrm{Tr}(\bar B_{6\mu}^* A^\mu B_6) + \mathrm{H.c.} + g_4 \mathrm{Tr}(\bar B_{6\mu}^* A^\mu B_{\bar 3}) + \mathrm{H.c.} \\
& + g_5 \mathrm{Tr}(\bar B_{6\nu}^* \gamma_\mu \gamma_5 A^\mu B_6^{*\nu}) + g_6 \mathrm{Tr}(\bar B_{\bar 3} \gamma_\mu \gamma_5 A^\mu B_{\bar 3}) \,,
\end{aligned} \tag{40}$$

where $B_{6\mu}^*$ is a Rarita–Schwinger vector-spinor field for a spin $\frac{3}{2}$ particle [13], and A_μ is the axial field introduced in (31).

4 Applications

Using the bag model wave functions constructed in the previous section, one can proceed to the calculation of the strong decay coupling constants ($g_1 \sim g_6$). However, the six coupling constants can be reduced to two independent ones by the heavy quark symmetry (HQS) [9, 10]. Furthermore, these two coupling constants are independent of the heavy mass.

Now we will begin with the model calculation for the coupling constants. The strong coupling constants describe soft pion emission. Applying PCAC, we can express a soft pion amplitude to a matrix element of the axial-vector current

$$\langle B' \pi^a\ (q) | B \rangle = \frac{q^\mu}{f_\pi} \langle B' | A_\mu^a | B \rangle \,. \tag{41}$$

Here we consider the strong decay $\Sigma_Q^{+1} \to \Lambda_Q \pi^+$ firstly. The Lagrangian equation (40) leads to the coupling

$$\mathcal{L}_{\Sigma_Q \Lambda_Q \pi} = \frac{g_2}{\sqrt{2} f_\pi} \bar\Sigma_Q^{+1} \gamma^\mu \gamma_5 \Lambda_Q \partial_\mu \pi^+ \,. \tag{42}$$

If we define $g_A^{\Sigma_Q \Lambda_Q}$ by

$$\begin{aligned}
\langle \Lambda_Q | A_\mu^1 - \mathrm{i} A_\mu^2 | \Sigma_Q^{+1} \rangle = {} & g_A^{\Sigma_Q \Lambda_Q} \left(q^2\right) \bar u \left(\Lambda_Q\right) \gamma_\mu \gamma_5 u \left(\Sigma_Q^{+1}\right) \\
& + \cdots \,,
\end{aligned} \tag{43}$$

where the unlisted terms vanish at $q = 0$. Combining (41) and (43) and comparing with (42), we obtain

$$g_2 = -g_A^{\Sigma_Q \Lambda_Q}(0) \tag{44}$$

The left hand side of (43) can be calculated in the MIT bag model, as is diagrammatically illustrated in Fig. 1. The $\Delta I_3 = -1$ transition can be described by the current $\widetilde{\Sigma}_3$ (choosing $\mu = 3$)

$$\begin{aligned}
\widetilde{\Sigma}_3 &= \int \mathrm{d}^3 r (A_3^1 - \mathrm{i} A_3^2) \\
&= \int \mathrm{d}^3 r b_{nlm}^{\dagger d} \bar u_{nlm}(r,t) \gamma_3 \gamma_5 u_{nlm}(r,t) b_{nlm}^u \,,
\end{aligned} \tag{45}$$

where the superscript $u(d)$ represents the $u(d)$ quark. Here the light quark states have $n = 1$, $l = 0$, and $m = 0$ (S-wave), because we deal with the B_6, B_6^*, and $B_{\bar 3}$ baryons. Therefore, we obtain

$$g_2 = -\mathcal{I} \left\langle \Lambda_Q \uparrow \middle| \widetilde{\Sigma}_3 \middle| \Sigma_Q^{+1} \uparrow \right\rangle \,, \tag{46}$$

where

$$\begin{aligned}
\mathcal{I} \equiv {} & 4\pi N_I N_F \int_0^R \mathrm{d} r r^2 \\
& \times \left[j_0(X_F\, \bar r) j_0(X_I\, \bar r) - \frac{1}{3} j_1(X_F\, \bar r) j_1(X_I\, \bar r) \right] ;
\end{aligned} \tag{47}$$

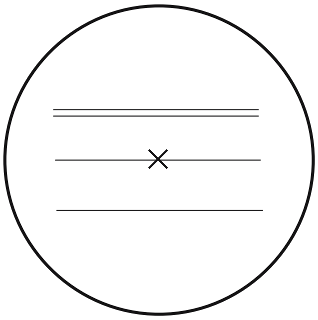

Fig. 1. Single particle transition in the bag. The *single* (*double*) *line* stands for a light (heavy) quark, and the *cross* represents an external current operator

we have $\bar{r} \equiv r/R$, and we have used the identities

$$\boldsymbol{\sigma} \cdot \widehat{r}\, \sigma_i\, \boldsymbol{\sigma} \cdot \widehat{r} = 2\, r_i \sigma \cdot \widehat{r} - \sigma_i ,$$
$$\int \mathrm{d}^3 r f(r) \widehat{r} (\boldsymbol{\sigma} \cdot \widehat{r}) = \frac{1}{3} \int \mathrm{d}^3 r f(r) \boldsymbol{\sigma} .$$

Equations (18) and (20) then give

$$\left\langle \Lambda_Q \uparrow \left| \widetilde{\Sigma}_3 \right| \Sigma_Q^{+1} \uparrow \right\rangle = -\frac{1}{2} \sqrt{\frac{1}{6}} (-4) \left\langle d \uparrow \left| \widetilde{\Sigma}_3 \right| u \uparrow \right\rangle = \sqrt{\frac{2}{3}} . \tag{48}$$

The decay width of the decay $\Sigma_Q^{+1} \to \Lambda_Q \pi$ is

$$\Gamma(\Sigma_Q \to \Lambda_Q \pi) = \frac{|\mathbf{p}|}{8\pi M_{\Sigma_Q}^2} g_{\Sigma_Q \Lambda_Q \pi}^2 \left[\left(M_{\Sigma_Q} - M_{\Lambda_Q} \right)^2 - M_\pi^2 \right] , \tag{49}$$

where $\mathbf{p}$ is the pion momentum in the c.m. system and where

$$g_{\Sigma_Q \Lambda_Q \pi} = \frac{M_{\Sigma_Q} + M_{\Lambda_Q}}{\sqrt{2} f_\pi} g_2 \tag{50}$$

is the Goldberger–Treiman relation.

Next, we consider the strong decay $\Sigma_Q^{*+1} \to \Lambda_Q \pi$. The Lagrangian equation (40) leads to the coupling

$$\mathcal{L}_{\Sigma_Q^* \Lambda_Q \pi} = \frac{g_4}{\sqrt{2} f_\pi} \bar{\Sigma}_{Q\mu}^{*+1} \Lambda_Q \partial^\mu \pi^+ . \tag{51}$$

We may take advantage of the constraints imposed by HQS [9, 10]:

$$g_4 = -\sqrt{3} g_2 . \tag{52}$$

The decay width of the decay $\Sigma_Q^{*+1} \to \Lambda_Q \pi$ is

$$\Gamma(\Sigma_Q^* \to \Lambda_Q \pi) = \frac{|\mathbf{p}|^3}{96\pi M_{\Sigma_Q^*}^2 M_{\Lambda_Q}^2} g_{\Sigma_Q^* \Lambda_Q \pi}^2 \times \left[\left(M_{\Sigma_Q^*} + M_{\Lambda_Q} \right)^2 - M_\pi^2 \right] , \tag{53}$$

and where

$$g_{\Sigma_Q^* \Lambda_Q \pi} = \frac{M_{\Sigma_Q^*} + M_{\Lambda_Q}}{\sqrt{2} f_\pi} g_4 \tag{54}$$

is the Goldberger–Treiman relation again.

The parameters we use as input to calculate $\mathcal{I}$ are the light quark masses $m_{u,d}$ and the radius of the bag, R. The current light quark masses in general are taken as $m_{u,d} = 0$. As to the radius R, $R_{\Sigma_c^{++}} = 0.945$ fm, $R_{\Sigma_c^{*++}} = 1.01$ fm, and $R_{\Lambda_c^+} = 0.914$ fm were given by [14] in the charm sector, and $R_{\Sigma_b^+} = 1.02$ fm, $R_{\Sigma_b^{*+}} = 1.04$ fm, and $R_{\Lambda_b^0} = 0.996$ fm were given by [15] in the bottom sector. They were obtained by fitting the mass spectrum in the respective sectors. Here we use the parameters $m_{u,d} = 0$ and $R = 1$ fm in (47) and obtain $\mathcal{I} = 0.653$. Then the strong decay widths in (49) and (53) can be estimated, and the numerical results in the charm sector are listed in Tables 2 and 3, respectively. We find that the results are all consistent with the experimental data. In addition, if the value of $m_{u,d}$ varies from 0 to 8 MeV and R varies from 0.8 fm to 1.2 fm, then for example the decay width $\Gamma(\Sigma_c^{++} \to \Lambda_c^+ \pi^+)$ varies just from 1.90 to 1.94 MeV. That is, these calculations are insensitive to the values of $m_{u,d}$ and R. We also present the experimental data and other theoretical calculations in the tables.

Finally, we use the same parameters to predict the relevant strong decay widths in the bottom sector. Because the neutral bottom baryons $\Sigma_b^{0(*)}$ have not been found yet, we resort to the following supposition to obtain $M_{\Sigma_b^{(*)0}}$: the mass differences ΔM_b among the bottom baryons Σ_b^+, Σ_b^0, and Σ_b^- come from two parts; one part is the quark mass difference $m_d - m_u$, and the other part is the electromagnetic Coulomb energy. Thus we may take the quark replacement $b \to s$ and observe the mass differences among Σ^+, Σ^0, and Σ^-. From [2], it is easy to find that

$$M_{\Sigma^0} \simeq \frac{1}{2} (M_{\Sigma^+} + M_{\Sigma^-}), \quad M_{\Sigma^{*0}} \simeq \frac{1}{2} (M_{\Sigma^{*+}} + M_{\Sigma^{*-}}) . \tag{55}$$

The errors are both smaller than 10^{-3}. Therefore, we may approximately use the similar equations

$$M_{\Sigma_b^0} \to \frac{1}{2} \left(M_{\Sigma_b^+} + M_{\Sigma_b^-} \right) ,$$
$$M_{\Sigma_b^{*0}} \to \frac{1}{2} \left(M_{\Sigma_b^{*+}} + M_{\Sigma_b^{*-}} \right) . \tag{56}$$

The results of the decay widths $\Gamma(\Sigma_b^{(*)} \to \Lambda_b \pi)$ are listed in Table 4.

5 Conclusion

In this paper we have presented a formalism to describe the chiral dynamics of the heavy baryons interacting with the Goldstone bosons. Furthermore, through PCAC, we have obtained the relevant strong interaction constants in the MIT bag model and then estimated the strong decay widths $\Gamma(\Sigma_{b,c}^{(*)} \to \Lambda_{b,c} \pi)$. The parameters appearing in this approach are the light quark masses $m_{u,d}$ and the bag radius R. On the one hand the current masses of the u and d quarks are both very small on the typical hadron energy scale and can be taken as zero, and on the other hand the

Table 2. Decay widths $\Gamma(\Sigma_c \to \Lambda_c \pi)$ (in units of MeV). Experimental data and other theoretical calculations are also shown

	$\Gamma(\Sigma_c^{++} \to \Lambda_c^+ \pi^+)$	$\Gamma(\Sigma_c^+ \to \Lambda_c^+ \pi^0)$	$\Gamma(\Sigma_c^0 \to \Lambda_c^+ \pi^-)$
Experiment [2]	2.23 ± 0.30	< 4.6 (CL = 90%)	2.2 ± 0.4
This work	1.90	2.20	1.87
CQM [16]	1.31 ± 0.04	1.31 ± 0.04	1.31 ± 0.04
CQM [17]	$2.025^{+1.134}_{-0.987}$		$1.939^{+1.114}_{-0.954}$
HHCPT [9, 10]	2.47, 4.38	2.85, 5.06	2.45, 4.35
HHCPT [18]	2.5	3.2	2.4
HHCPT [19]			1.94 ± 0.57
HHCPT [20]	input	2.6 ± 0.4	2.2 ± 0.3
LFQM [21]	1.64	1.70	1.57
RTQM [22, 23]	2.85 ± 0.19	3.63 ± 0.27	2.65 ± 0.19
NRQM [24]	$2.41 \pm 0.07 \pm 0.02$	$2.79 \pm 0.08 \pm 0.02$	$2.37 \pm 0.07 \pm 0.02$

Table 3. Decay widths $\Gamma(\Sigma_c^* \to \Lambda_c \pi)$ (in units of MeV). Experimental data and other theoretical calculations are also shown

	$\Gamma(\Sigma_c^{*++} \to \Lambda_c^+ \pi^+)$	$\Gamma(\Sigma_c^{*+} \to \Lambda_c^+ \pi^0)$	$\Gamma(\Sigma_c^{*0} \to \Lambda_c^+ \pi^-)$
Experiment [2]	14.9 ± 1.9	< 17 (CL = 90%)	16.1 ± 2.1
This work	14.7	15.2	14.5
CQM [16]	20	20	20
HHCPT [18]	25	25	25
HHCPT [20]	16.7 ± 2.3	17.4 ± 2.3	16.6 ± 2.2
LFQM [21]	12.84		12.40
RTQM [22, 23]	21.99 ± 0.87		21.21 ± 0.81
NRQM [24]	$17.52 \pm 0.74 \pm 0.12$	$17.31 \pm 0.73 \pm 0.12$	$16.90 \pm 0.71 \pm 0.12$

Table 4. The predictions of the strong decay widths $\Gamma(\Sigma_b^{(*)} \to \Lambda_b \pi)$ (in units of MeV)

$\Gamma(\Sigma_b^+ \to \Lambda_b^0 \pi^+)$	$\Gamma(\Sigma_b^0 \to \Lambda_b^0 \pi^0)$	$\Gamma(\Sigma_b^- \to \Lambda_b^0 \pi^-)$	$\Gamma(\Sigma_b^{*+} \to \Lambda_b^0 \pi^+)$	$\Gamma(\Sigma_b^{*0} \to \Lambda_b^0 \pi^0)$	$\Gamma(\Sigma_b^{*-} \to \Lambda_b^0 \pi^-)$
4.35	5.65	5.77	8.50	10.20	10.44

bag radius $R = 1$ fm was obtained by fitting the mass spectrum in the charm [14] and bottom [15] sectors and averaging them. In addition, we also separately vary $m_{u,d}$ from 0 to 8 MeV and R from 0.8 to 1.2 fm and find that the decay widths are insensitive to these two parameters. Based on the numerical results being all consistent with the experimental data in the charm sector, we give our predictions for the strong decay widths $\Gamma(\Sigma_b^{(*)} \to \Lambda_b \pi)$. In spite of the bottom baryons $\Sigma_b^{(*)0}$ not having been found, we observed that the masses of the neutral baryons $M_{\Sigma^{(*)0}}$ almost are equal to the average of the ones of the charged baryons $M_{\Sigma^{(*)+}}$ and $M_{\Sigma^{(*)-}}$, and then we reasoned in analogy to the masses of the neutral bottom baryons $M_{\Sigma_b^{(*)0}}$. We expect that the deviations of this assumption and the predicted strong decay widths are all small for the future experimental data.

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